OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4767:** Statistics 2

Mark Scheme for January 2012

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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1. **Annotations and abbreviations**

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. **Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand**

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

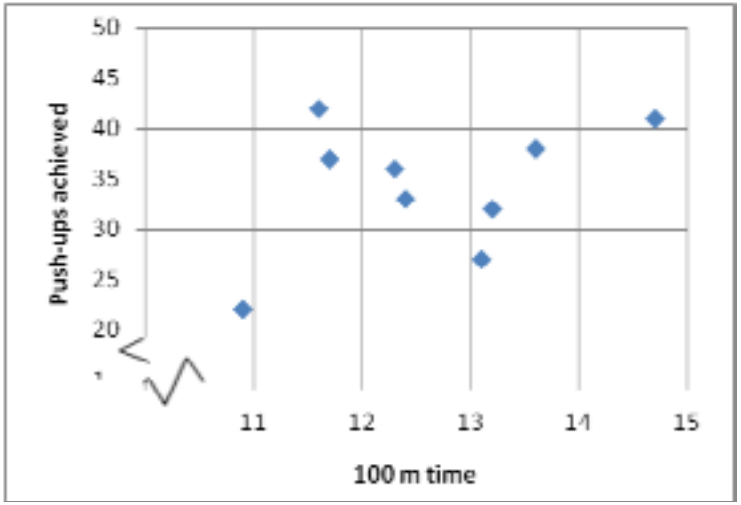
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be

the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(i)			G1 G2,1,0 [3]	G1 For axes suitably labelled with some indication of **linear** scale provided. G2 for points plotted correctly. G1 if 8 points plotted correctly. G0 if two or more incorrectly plotted/omitted points. Special Case SC1 for points visibly correct on axes where no indication of scale has been provided. Allow axes reversed

1 (ii)

Athlete	A	B	C	D	E	F	G	H	I
100 metre time	13.2	11.6	10.9	12.3	14.7	13.1	11.7	13.6	12.4
Push ups achieved	32	42	22	36	41	27	37	38	33
Rank 100m	7	2	1	4	9	6	3	8	5
Rank Push-ups	3	9	1	5	8	2	6	7	4
d	4	–7	0	–1	1	4	–3	1	1
d^2	16	49	0	1	1	16	9	1	1

M1 For ranking (allow all ranks reversed for either or both categories)
NB No ranking or re-allocation of pairs scores 0/5

M1 For d^2

$\Sigma d^2 = 94$

A1 For Σd^2

$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6\times 94}{9\times 80} = 1 - 0.783$

M1 M1 for method for r_s used

= 0.217 (to 3 s.f.) [*allow* 0.22 to 2 s.f.]

A1 A1 f.t. for $|r_s| < 1$
Allow 13/60
or $r_s = 1 - 1.217 = -0.217$ with reversed ranks

Push-up times ranked from highest (1st) to Lowest (9th) gives $\Sigma d^2 = 146$ which leads to -0.217. Allow both A marks.

	A	B	C	D	E	F	G	H	I
100m	7	2	1	4	9	6	3	8	5
Push up	7	1	9	5	2	8	4	3	6
d	0	1	-8	-1	7	-2	-1	5	-1
d^2	0	1	64	1	49	4	1	25	1

[5]

1	**(iii)**		H_0: no association between 100m time and number of push-ups achieved in the population of long distance runners	B1	B1 for H_0 in context (not x & y)	
			H_1: some association between 100m time and number of push-ups achieved in the population of long distance runners	B1	B1 for H_1 in context (not x & y) SC1 for both correct but no context provided	
				B1	B1 for population SOI NB H_0 H_1 <u>not</u> ito ρ Do not condone the use of the word 'correlation' in place of 'association'. Population' should be mentioned to award B1, unless clear, unambiguous alternative wording is used.	
			Two tail test critical value at 5% level is 0.7000	B1	B1 for ±0.7000 (or ±0.6000 only if H_1 indicates a 1-tailed test is intended)	
			Since 0.217 < 0.7000, there is insufficient evidence to reject H_0,	M1	M1 for sensible comparison with c.v leading to a conclusion seen, provided $\|r_s\| < 1$ NOTE The comparison can be in the form of a diagram as long as it is clear and unambiguous. Sensible comparison: e.g. – 0.217 > – 0.7000 is 'sensible' whereas – 0.217 < 0.7000 is 'not sensible'. Allow -0.7000 < 0.217 < 0.7000 Reversed inequality sign e.g. 0.217 > 0.7000 etc gets max M1 A0. Also, if the c.v. comes from the p.m.c.c. table (0.6664 for 2-tailed test and 0.5822 for a 1-tailed test) award max M1 A0.	

			i.e. conclude that there is not enough evidence to show association between the 100m times and the number of push-ups achieved.	A1 **[6]**	A1ft for correct conclusion in context. Follow through their r_s NOTE 2-tailed test with correct c.v. must be used to award final A1. Use of a 1-tailed test: max B1 B0 B1 B1 (for 0.6000) M1 A0. i.e. max 4/6. Where hypotheses are reversed, lose first two B1 marks and final A1. max 3/6	
1	**(iv)**		It is appropriate to carry out a hypothesis test based on the product moment correlation coefficient when the **underlying** population has a **bivariate Normal distribution.** The scatter diagram does not appear to be roughly elliptical so the Spearman coefficient is more appropriate	E1 E1 E1dep **[3]**	Do not accept 'both Norrnally distributed' Allow reasonable alternatives e.g. in this case, one variable is discrete so pmcc invalid. E1 dependent on previous E1	
2	**(i)**		**Errors** have a **uniform average rate** of occurrence and occur randomly and independently	E1 E1 **[2]**	E1 must refer to 'errors' not 'events', 'data' or 'conditions'. Condone 'constant/fixed average/mean rate/per page' but not 'constant average', 'constant rate' or 'uniform rate', etc. Allow large n and small p if both defined E1 for randomly **and** independently If 'errors' not referred to then SC1 if otherwise correct. Condone 'the number of errors'	

2	**(ii)**	(*A*)	$P(X = 1) = e^{-0.85}\frac{0.85^1}{1!} = 0.3633$	M1 A1	M1 for attempt to find $P(X = 1)$ either by Poisson p.d.f. or use of tables. A1 CAO 3s.f. for answers which round to 0.363 www NOTE If $P(X \leq 1)$ used for final answer, award M0A0. Interpolation gives 0.79065 – 0.42795 = 0.3627	
		(*B*)	$P(X \geq 2) = 1 - P(X \leq 1) = 1 - e^{-0.85}\frac{0.85^0}{0!} - e^{-0.85}\frac{0.85^1}{1!}$ $= 1 - 0.4274 - 0.3633 = 0.2093$	M1 M1 A1 **[5]**	M1 for method for $P(X = 0)$ M1 for correct structure used A1 CAO 3s.f for answers which round to 0.209 Allow 0.2094 if interpolation used. [Interpolation gives 0.42795 for $P(X = 0)$ and 0.20935 for $P(X \leq 1)$]	
2	**(iii)**		New $\lambda = 10 \times 0.85 = 8.5$ P(Exactly 10 in 10 pages) = 0.7634 – 0.6530 = 0.1104 Or $= e^{-8.5}\frac{8.5^{10}}{10!} = 0.1104$	B1 M1 A1 **[3]**	B1 for 8.5 M1 for $P(X = 10)$ calculation using $\lambda = 8.5$ CAO Allow 0.110 and 0.11 www Award M1 only if $\lambda = 8.5$ used	

2	(iv)		So P(k – 1 or less in 10 pages) > 99% From tables P(X ≤ 15) = 0.9862, P(X ≤ 16) = 0.9934 P(X ≥ 16) = 1 - P(X ≤ 15) = 0.0138 > 1% P(X ≥ 17) = 1 - P(X ≤ 16) = 0.0066 < 1% P(k or more in 10 pages) < 1% means k – 1 = 16, k = 17	M1 A1 A1 A1 **[4]**	M1 for P(X ≤ k – 1) > 0.99 seen, or evidence of a search for values > 0.99 from cumulative Poisson tables seen. A1 for finding either one of 0.9862 and 0.9934 (or either one of 0.0138 and 0.0066) A1 for both (3s.f.) A1 CAO for k = 17 SC1 for evidence of a search for values > **0.9** from cumulative Poisson tables seen. Or for k = 17with no supporting evidence seen.	
2	(v)		Mean number in 30 pages = 30 × 0.85 = 25.5 Using Normal approx. to the Poisson, X ~ N(25.5, 25.5) $P(X \le 30) = P\left(Z \le \frac{30.5-25.5}{\sqrt{25.5}}\right)$ = P(Z < 0.9901) = Φ(0.9901) = 0.8389	 B1 B1 B1 M1 A1 **[5]**	 For Normal approx attempted.(SOI) For correct parameters (SOI) For correct continuity correction. For correct structure with their parameters CAO **(**Do not FT wrong or omitted CC) Allow 0.839	
3	(i)		$P(1100 < X < 1200) = P\left(\frac{1100-1100}{\sqrt{2000}} < Z < \frac{1200-1100}{\sqrt{2000}}\right)$ = P(0< Z < 2.236) = Φ(2.236) – 0.5 = 0.9873 – 0.5 = 0.4873	M1 M1 A1 **[3]**	For for standardising M0 if 'continuity correction' applied For for correct structure A1 CAO do not allow 0.4871, 0.48713, 0.48745 or 0.4875	

3	(ii)		Use Normal approx with $\mu = np = 100 \times 0.4873 = 48.73$ $\sigma^2 = npq = 100 \times 0.4873 \times 0.5127 = 24.98$ $P(X \leq 40) = P\left(Z \leq \frac{40.5-48.73}{\sqrt{24.98}}\right)$ $= P(Z \leq -1.647) = 1 - \Phi(1.647) = 1 - 0.9502$ $= 0.0498$	B1 B1 B1 M1 A1 **[5]**	An appropriate Normal approximation must be used. B1 B1 for μ & σ^2 ft their answer to part (i) provided that a normal approximation is appropriate from their part (i). B1 for continuity correction 40.5 M1 correct structure using appropriate Normal approximation CAO 3s.f. NOTE Using B(100, 0.4873) gives 0.0494. which gets 0/5 SC If *p* small enough to justify a Poisson approximation, e.g. 0.05, then B1 for Poisson used, B1 ft for parameter, M1 for structure,M1 attempt at summation, A0	
3	(iii)		From tables $\Phi^{-1}(0.01) = -2.326$ $\frac{k-1100}{\sqrt{2000}} = -2.326$ $k = 1100 - 2.326 \times \sqrt{2000}$ $k = 996$	B1 M1 A1 **[3]**	±2.326 seen M1 correct equation as seen or equivalent CAO Allow 996.0	

3	**(iv)**		H_0: $\mu = 7000$; H_1: $\mu \neq 7000$	B1 B1	B1 For use of 7000 in hypotheses B1 For correct hypotheses given in terms of μ (not p or x, etc. unless letter used is clearly defined as population mean.) If hypotheses are reversed lose second B1 and final A1
			Where μ denotes the population mean lifetime of low energy bulbs	B1	B1 for definition of μ.
			Test statistic $= \dfrac{6972-7000}{100/\sqrt{25}} = \dfrac{-28}{20} = -1.4$	M1 A1	M1 calculation of test statistic with a divisor of $100/\sqrt{25}$. Condone numerator reversed. CAO for – 1.4 Allow +1.4 for A1 only if this is later compared with +1.645
			Lower 10% level 2 tailed critical value of $z = -1.645$	B1	For –1.645 Must be negative unless it is clear that absolute values are being used. NB: FT a 1-tail test (c.v. = –1.282)
			–1.4 > –1.645 so not significant. There is not sufficient evidence to reject H_0 There is insufficient evidence to conclude that the manufacturer is wrong.	M1 A1	M1 for a sensible comparison leading to a conclusion. For correct conclusion in context **FT only** their test statistic **if c.v. correct and both M marks earned.** <u>Critical Value Method</u> 7000 – 1.645 × 100 ÷ √25 gets M1B1 = 6967.1 gets A1 6972 > 6967.1 gets M1for sensible comparison A1 still available for correct conclusion in words & context <u>Confidence Interval Method</u> CI centred on 6972 + or – 1.645 × 100 ÷ √25 gets M1 B1 = (6939.1, 7004.9) gets A1 contains 7000 gets M1 A1 still available for correct conclusion in words & context

				[8]	Probability Method Finding P(sample mean < 6972) = 0.0808 gets M1 A1 B1 0.0808 > 0.05 gets M1 for a sensible comparison if a conclusion is made. 0.0808 < 0.10 gets M1 A0 unless using one-tailed test A1 still available for correct (one-tailed) conclusion in words & context. Condone P(sample mean > 6972) = 0.9192 for M1 but only allow A1 B1 if later compared with 0.95, at which point the final M1and A1 are still available One-tailed test Max B1 B0 B1 M1 A1 B1 (for cv = -1.282) M1 A0
4	**(i)**		Expected frequency = 29/305 × 98 = 9.3180	M1 A1 **[2]**	M1 for row total × column total ÷ grand total ***NB Answer given***
4	**(ii)**		Contribution = $(17 – 9.3180)^2 / 9.3180$ = 6.3332	M1 A1 **[2]**	M1 for valid attempt at $(fo – fe)^2 \div fe$ ***NB Answer given***
4	**(iii)**		H_0: no association between place and species. H_1: some association between place and species. Test statistic $X^2 = 136.185$ Refer to X_4^2 Critical value at 1% level = 13.28 Result is significant There is (strong) evidence to suggest association between place and species.	B1 M1 A1 B1 B1 B1 E1 **[7]**	B1 for hypotheses in context NB if H_0 H_1 reversed, or 'correlation' mentioned, do not award first B1 or final E1 M1 for sum of contributions A1 for test statistic. Allow 136.1847 For 4 deg of freedom (seen) CAO For cv – no FT from here if wrong/omitted For significant E1 for correct conclusion in context

4	**(iv)**		Farm Contribution of 11.6539 implies that there are (far) fewer thrushes than expected Also more finches than expected and more tits. Garden Contribution of 60.7489 implies that there are far more thrushes than expected. Contribution of 20.5201 implies that there are (far) fewer tits than expected. Also fewer finches than expected. Woodland Contribution of 19.2192 implies that there are (far) fewer thrushes than expected. Contribution of 9.5969 implies that there are (far) more tits than expected. Number of finches is as expected.	E2,1,0 E2,1, 0 E2,1, 0 **[6]**	**For each place -** award E2 for an explicit reference to a value from the contributions table with a correct* corresponding comment, provided that no further incorrect statements are made regarding that place. *See table below Wording needs to be clear. Do not accept 'different to expected' or 'positive/negative association' Award E1 **either** for an explicit/implicit reference to a value from the contributions table with a correct corresponding comment accompanied by one or more incorrect comment for that place Allow alternative wording. e.g. 'more thrushes were expected' in place of 'there were fewer thrushes than expected'	

	Farm	Garden	Woodland
Thrushes	11.6539 (far) fewer than expected	60.7489 **Far** more than expected	19.2192 (far) fewer than expected
Tits	2.0017 As expected/ more than expected	20.5201 (far) fewer than expected	9.5969 (far) more than expected
Finches	6.3332 (far) more than expected	6.1108 fewer than expected	0.0000 as expected

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